Modeling the Effect of Measures to Limit the Spread of Infectious Diseases

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Introduction

Infectious diseases have posed a threat throughout human history. Despite the rapid development of medicine and technology, they continue to be a serious problem today. The civilized way of life weakens the immune system of the human body, and frequent trips outside the local communities create a precondition for the spread of viruses. After the first outbreak of a new infection, it was soon spread to all parts of the world. Medicine does not always manage to find a solution to the problem immediately, without side effects on health. Often, to deal with another virus, it is enough to introduce measures to limit its spread, plus means to increase the body's natural resistance. Such is, for example, the strategy used during each winter period for another influenza virus strain. Enhanced hygiene measures through the use of various disinfectants in public places, limiting close contact with infected people are logical and time-proven techniques. Extraordinary flu holidays for students, absence from work for 3-5 days for people who have shown symptoms are approved and accepted by society. They bring inconvenience, but do not drastically change everyday life for a long period of time, do not lead to large losses. Naturally, hypothetically, if every member of our society isolates itself, viruses will not be able to spread so easily. But it will also no longer be a society. The solution is probably to maintain a reasonable balance in restrictive measures for the spread of a virus, which also depends on the likelihood of cure with natural or medicinal remedies. A 14-day quarantine period for transcontinental travelers could also be established as a practice.

This article aims to model and study the effect of the strength, time and duration of the restrictive measures for the spread of an infectious disease. The inconveniences, economic losses and gaps in education are the price that society pays to prevent the spread of the virus. It is important that restrictive measures are observed as soon as possible, at the most appropriate time, in order to have minimal negative consequences for society, and at the same time to be effective against the spread of the virus.

Methods and Models

Compartmental models are widely used among epidemiologists to simulate disease dynamics. The origin of such models is the early 20th century ([1]). Depending on the disease, the compartments can be susceptible (S), exposed (E), infectious (I), or recovered (R). Some infections, for example, those from the common cold, influenza and the recent one COVID-19, do not confer any long-lasting immunity. Such infections do not give immunity upon recovery from infection, and surviving individuals become susceptible to the disease again. The notation SIS is used to describe a disease with no immunity against reinfection, to indicate that the passage of individuals is from the susceptible class to the infective class and then back to the susceptible class. Another commonly used model is SIR (Susceptible – Infectious -

Recovered), which suggests that re-infection with the disease is not possible, i.e. the patient dies or recovers, developing immunity against future infection.

Although the new world-famous virus COVID-19 appeared only at the end of 2019, there are already publications using the SIS and SIR models to simulate its spread ([2] – [8]). Recently, cases of people with established re-infection with COVID-19 have been reported in the media for only a few months after successfully recovering from the initial infection. Therefore, in this article we will use the SIS distribution model as a basis.

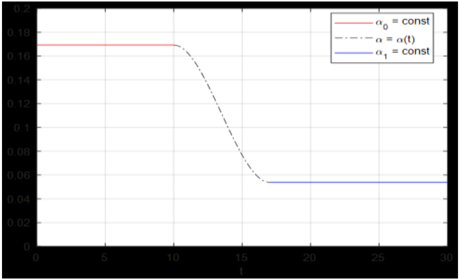
Let $I(t)$ denote the proportion of infected persons at the time t. Then $I(t)$ satisfies the differential equation (see [6])

$$\frac{dI(t)}{dt} = \alpha I(1 - I) - \gamma I. \qquad (1)$$

Here α and γ are the disease transmission rate and the recovery rate.

In the classical SIS model, α and γ are assumed to be positive constants. However, after the introduction of *measures* to *limit* the *spread* of the *virus* (MLSV), the value of α should decrease, not suddenly, but gradually until the MLSV are observed by all members of the population. In the next Section, we apply numerical experiments assuming that the transmission rate α is a monotonically decreasing function of time.

Assume that, during an initial period $[0, t_0)$, no MLSV have been introduced and the value of the transmission rate is α_0. Then at the time t_0 the official implementation of the measures begins and from time t_1 the measures are applied by all, which reduces the value of the transmission rate to α_1 (Fig. 1).



In the interval $[t_0, t_1)$ the transmission rate α can be modelled as a function of time $\alpha = \alpha(t)$ with a polynomial $P_n(t)$, for which the following conditions are met

$$\left| \begin{array}{l} P_n(t_0) = \alpha_0 \\ P_n(t_1) = \alpha_1 \\ P_n'(t_0) = 0 \\ P_n'(t_1) = 0 \end{array} \right. . \qquad (2)$$

They would guarantee a continuous first derivative for $\alpha(t)$ in the interval $[0, t^*]$, where t^* is the moment in which the measures are stopped. We will assume that t^* is determined by the condition "reaching a given proportion I^* of infected", I^* is a constant, $I^* \in (0, I(t_0))$.

The simplest polynomial that would satisfy (2) is of degree three, $P_3(t) = b_3 t^3 + b_2 t^2 + b_1 t + b_0$, with coefficients

$$b_3 = \frac{2(\alpha_0 - \alpha_1)}{(t_1 - t_0)^3}, \quad b_2 = -\frac{3(t_1 + t_0)(\alpha_0 - \alpha_1)}{(t_1 - t_0)^3},$$

$$b_1 = \frac{6t_1 t_0(\alpha_0 - \alpha_1)}{(t_1 - t_0)^3}, \quad b_0 = \frac{-\alpha_1 t_0^3 + 3\alpha_1 t_0^2 t_1 - 3\alpha_0 t_0 t_1^2 + \alpha_0 t_1^3}{(t_1 - t_0)^3}. \tag{3}$$

The economic losses suffered from the imposed restrictions in the interval $[t_0, t^*]$ depend on the severity of the measures, i.e. on the difference $\alpha_0 - \alpha(t)$ and on the duration of their application - $t^* - t_0$.

In his article ([6]), Kahale' assumes that $t_0 = t_1$ and uses the model

$$L = c(\alpha_0 - \alpha_1)(t^* - t_0)$$

for the total economic losses, giving a maximally simple, linear and directly proportional dependence. In this case L can also be written as

$$L = c\int_{t_0}^{t^*}(\alpha_0 - \alpha_1)dt\,.$$

At a time-dependent transmission rate α then we will have

$$L = c\int_{t_0}^{t^*}\left(\alpha_0 - \alpha(t)\right)dt = c\left[\int_{t_0}^{t^*}\alpha_0 dt - \int_{t_0}^{t_1}P_3(t)dt - \int_{t_1}^{t^*}\alpha_1 dt\right]. \qquad (4)$$

After performing transformations in (4), using equations (3) for the coefficients of the polynomial $P_3(t)$, the following representation is reached:

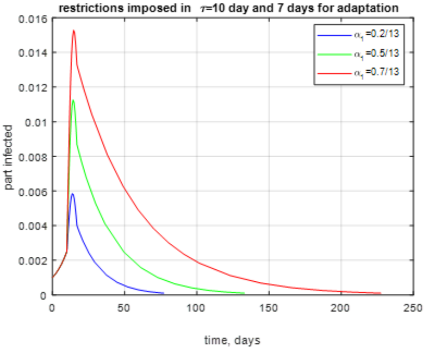
$$L = c(\alpha_0 - \alpha_1)\left(t^* - \frac{t_0 + t_1}{2}\right).$$

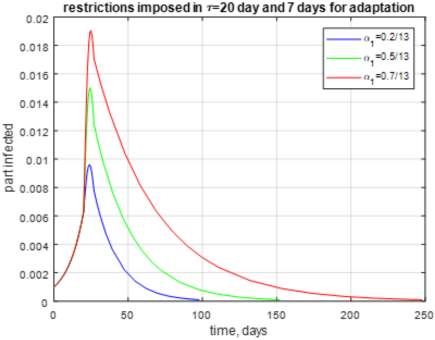
Numerical results

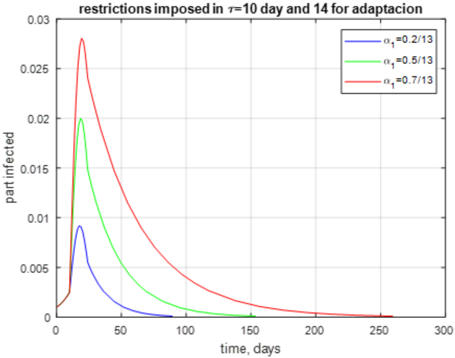
In the general case, when α and γ are not constants, the differential equation (1) is unsolvable in quadratures. The solution must be found numerically. Matlab's ode113 solver was used, implementing a multi-step Adams-Bashford-Multan method with a variable order of accuracy. All numerical experiments were performed at an initial proportion of infected $I(0) = 0.001$, a recovery rate $\gamma = 1/13$ and reaching a proportion of $I^* = 0.0001$ to drop the restrictive measures. For the transmission rate at the beginning α_0 we assume value $\alpha_0 = 2.2/13 = 0.1692$, and for the constant c – c=1. Similar parameters are considered in [6].

All results regarding the losses L and the moment of termination of the constraints t^* depend on the changes of the moment t_0 at which the constraints are introduced, the adaptation time $t_1 - t_0$, as well as on the value of α_1.

Figures 2 - 5 reflect the part of the infected over time, with various restrictive measures, at a fixed moment in which they are imposed, as well as with a fixed duration of adaptation to them. It can be seen that the proportion of infected is highly sensitive depending on the size of the measures. The stricter the MLSV measures, the faster they reach the point t^* where they are dropped.







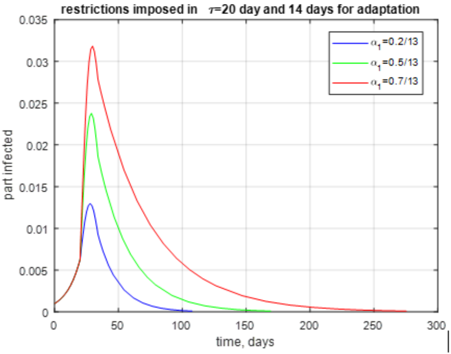
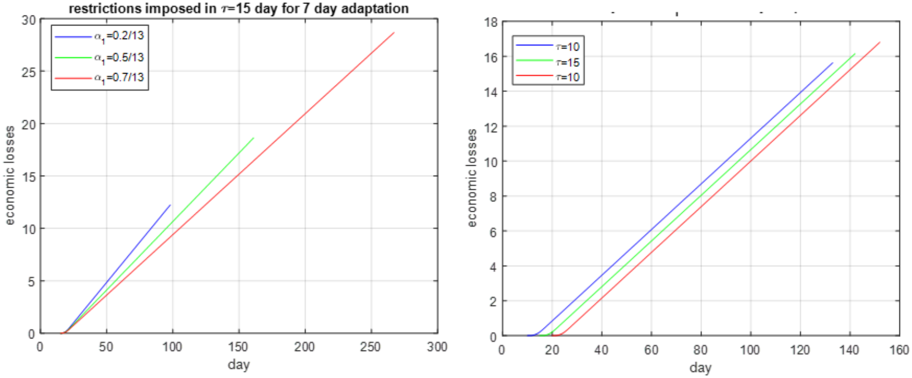


Fig. 6 reflects the total economic losses over time with various restrictive measures and at a fixed point in time in which they are imposed. Both in the part of the infected and in the losses there is a strong sensitivity to the size of the measures taken. Again, stricter measures for social distancing lead to a strong reduction in losses.

Figure 7 shows the economic losses over time at a fixed size of the restrictive measures, but introduced at different times. Losses increase linearly both depending on the length of the quarantine period and the time at which it is imposed.



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Thank you for the attention!